Digital Circuits

ECS 371

Dr. Prapun Suksompong

prapun@siit.tu.ac.th

Lecture 6

Office Hours:
BKD 3601-7
Monday 1:30-3:30
Tuesday 10:30-11:30

ECS371.PRAPUN.COM

Announcement

- Need to do something about the office hours.
- The old office hours
  - Monday 1:30-3:30PM: Conflict with ITS325, MAS210
  - Tuesday 10:30-11:30AM: Conflict with GTS211
- Let's add
  - **Monday 9:00-10:30**
- I'm not limited to these time slots.
  - Usually in my office (BKD3601-7) from 8AM-5PM

Today

- Some of us participate in the SIIT day activities.
- So, the lecture today will contain no new material.
  - We will do a lot of examples.
- These sides will be posted on the course web site later today.
  - Some of them are the same as what you have as hardcopy

- At 9:30, we will have attendance check
  - For those participate in SIIT D.A., Provide evidence to get checked.

Example

The associative law for addition is normally written as

a. $A + B = B + A$ ← Commutative

b. $(A + B) + C = A + (B + C)$ ← Associative

c. $AB = BA$

d. $A + AB = A$ ← Covering

Example

The Boolean equation $AB + AC = A(B + C)$ illustrates

a. the distribution law

b. the commutative law $\leftarrow X+Y = Y+X$, $XY = YX$

c. the associative law $\leftarrow X+(Y+Z) = (X+Y)+Z$

d. DeMorgan's theorem

Example

The Boolean expression $A \cdot 1$ is equal to

a. A

b. B

c. 0

d. 1

Example

The Boolean expression $A + 1$ is equal to

a. A

b. B

c. 0

d. 1

Example

The Boolean equation $AB + AC = A(B + C)$ illustrates

a. the distribution law

b. the commutative law

c. the associative law

d. DeMorgan's theorem

Review: Three Useful Rules

These rules do not exist in elementary algebra

1. $(A+B)(A+C) = A+BC$

2. $A + AB = A$

$= A\cdot 1 + A\cdot B$

$= A(1+B) = A\cdot 1 = A$

Examples:

$$\boxed{XY} + \boxed{XY}Z = \boxed{XY}$$

$$W\overline{X}Y + WY = WY$$

$= \overline{X}\boxed{WY} + \boxed{WY}$

$= \Box = WY$

3. $A + \overline{A}B = A + B$

$= A + AB + \overline{A}B$

$= A + B(A+\overline{A}) = A + B\cdot 1$

$= A + B$

Examples:

$$\overline{A} + AB = \overline{A} + B$$

$$XY + \overline{XY}Z = XY + Z$$

$$\boxed{\overline{A}} + \boxed{A}(\overline{B}+\overline{C}) = \overline{A} + \overline{B} + \overline{C}$$

Principle of Duality

Any theorem or identity remains true if $0 \leftrightarrow 1$

Example: $\bullet \leftrightarrow +$

$$X + 1 = 1 \qquad X + \overline{X} = 1$$

$$X \cdot 0 = 0 \qquad X \cdot \overline{X} = 0$$

Caution: $X + (X \cdot Y) = X \rightarrow X \cdot (X + Y) = X$

- Parenthesize an expression fully before taking its dual!

$\rightarrow (X \cdot Y) + (X \cdot Z) = X \cdot (Y + Z) \quad (X + Y) \cdot (X + Z)$

$\rightarrow (X + Y) \cdot (X + Z) = X + Y \cdot Z$

factoring

Duality Principle in Action

(T1)	$X + 0 = X$	(T1′)	$X \cdot 1 = X$	(Identities)
(T2)	$X + 1 = 1$	(T2′)	$X \cdot 0 = 0$	(Null elements)
(T3)	$X + X = X$	(T3′)	$X \cdot X = X$	(Idempotency)
(T4)	$(X')' = X$			(Involution)
(T5)	$X + X' = 1$	(T5′)	$X \cdot X' = 0$	(Complements)

(T6)	$X + Y = Y + X$	(T6′)	$X \cdot Y = Y \cdot X$	(Commutativity)
(T7)	$(X + Y) + Z = X + (Y + Z)$	(T7′)	$(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)$	(Associativity)
(T8)	$X \cdot Y + X \cdot Z = X \cdot (Y + Z)$	(T8′)	$(X + Y) \cdot (X + Z) = X + Y \cdot Z$	(Distributivity)
(T9)	$X + X \cdot Y = X$	(T9′)	$X \cdot (X + Y) = X$	(Covering)
(T10)	$X \cdot Y + X \cdot Y' = X$	(T10′)	$(X + Y) \cdot (X + Y') = X$	(Combining)

Example

Using Boolean algebra, simplify

$$\underbrace{BD+B(D+E)}_{BD+\cancel{BD}+BE}+\underbrace{\overline{D}(D+F)}_{\underbrace{\overline{D}D}_{0}+\overline{D}\cdot F}$$

$$= BD + BE + \overline{D}F \leftarrow \text{SOP}$$

$$\left(= B(D+E) + \overline{D}F\right)$$

Example

Using Boolean algebra, simplify

space

$$\overline{A}\,\overline{B}C + \underbrace{\overline{(A+B+\overline{C})}}_{\leftarrow \text{ DeMorgan's Theorem}} + \overline{A}\,\overline{B}\,\overline{C}D$$

$$= \bar{A}\bar{B}C + \cancel{\bar{A}\cdot\bar{B}\cdot C} + \bar{A}\bar{B}\bar{C}D$$

$$= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D$$

$$= \bar{A}\bar{B}(C + \bar{C}D) = \bar{A}\bar{B}(C+D) = \bar{A}\bar{B}C + \bar{A}\bar{B}D$$

Example

Show that

$$(A+B)(A+C)(A+D) = A+BCD$$

$$= (A+BC)(A+D)$$

$$= (A+\underset{BC}{\triangle})(A+\underset{D}{\square})$$

$$= A + \triangle \cdot \square = A + BCD$$

\#

Example

Using Boolean algebra, simplify

$$\left(B + BC\right)\left(B + \overline{B}C\right)\left(B + D\right)$$

$$= B + \left(BC\overline{B}CD\right)$$

$$0$$

$$= B$$

Example

Using Boolean algebra, simplify

$$\underbrace{ABCD + AB\left(\overline{CD}\right)} + \left(\overline{AB}\right)CD$$

$$AB\left(\underbrace{CD + \overline{CD}}_{1}\right)$$

$$AB \qquad + \quad \overline{AB}\,CD$$

$$= AB + CD$$

Example

Using Boolean algebra, simplify

$$ABC\left(AB+\underbrace{\overline{C}\left(BC+AC\right)}\right)$$

$$\underbrace{\overline{C}\cdot C}_{0}\cdot(B+A)$$

$$= ABC\cdot AB = ABC$$

(Quiz # 1 IT)

Example

K-map

Using Boolean algebra, simplify

$$A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

$$\bar{A}BC + \bar{B}\left(\underbrace{AC + \bar{A}C}_{C(\underbrace{A+\bar{A}}_{1})} + \underbrace{\bar{A}\bar{C} + A\bar{C}}_{\bar{C}(\underbrace{\bar{A}+A}_{1})}\right)$$

$$= \bar{A}BC + \bar{B}(\underbrace{C + \bar{C}}_{1}) = \bar{A}BC + \bar{B} = B(\bar{A}C) + \bar{B}$$

$$= \bar{B} + \bar{A}C$$

Example

HW # 2 already posted

Using Boolean algebra, simplify

$$\overline{A} + A\overline{B} + AB\overline{C}$$

$$\underbrace{\overline{A} + A\overline{B}}$$

$$\overline{A} + \underbrace{\overline{B} + AB\overline{C}}$$

$$= \overline{B} + B(A\overline{C})$$

$$= \overline{B} + A\overline{C}$$

$$= \underbrace{\overline{A} + \overline{B} + A\overline{C}} = \overline{B} + \overline{A} + \overline{C}$$

$$= \overline{A} + \overline{B} + \overline{C} = \overline{A \cdot B \cdot C}$$

Product Term

A single literal or a product of two or more literals.

Example: $A\cdot\overline{B}\cdot C$

$A\cdot C$

A

$A\cdot\overline{B}\cdot C\cdot D$

$\overline{A}\cdot\overline{B}\cdot\overline{C}$

Caution:

$\overline{A\cdot B\cdot C}$ is not a product term.

Q: When does $A\cdot\overline{B}\cdot C=1$?

$A=1$

$\overline{B}=1$

$C=1$

$\boxed{\begin{array}{l} A=1 \\ B=0 \\ C=1 \end{array}}$

$(A,B,C)=(1,0,1)$

Example

Find the value of X for all possible values of the variables when

$$\begin{array}{ccccc} 101 & 011 & 001 & 000 & 100 \\ X = ABC & + \overline{A}BC & + \overline{A}\overline{B}C & + \overline{A}\overline{B}\overline{C} & + A\overline{B}\overline{C} \end{array} \leftarrow$$

Standard sum of products

	A	B	C						
1	0	0	0	1	0	0	0	1	0
1	0	0	1	1	0	0	1	0	0
0	0	1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	0	0	0
1	1	0	0	1	0	0	0	0	1
1	1	0	1	1	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0

Example

Find the value of X for all possible values of the variables when

$$X = \overset{10}{A\overline{B}} + \overset{111}{ABC}$$

A	B	C	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1