

Digital Circuits

ECS 371

Dr. Prapun Suksompong

prapun@siit.tu.ac.th

Lecture 6

Office Hours:

BKD 3601-7

Monday 1:30-3:30

Tuesday 10:30-11:30

ECS371.PRAPUN.COM

Announcement

- Need to do something about the office hours.
 - The old office hours
 - Monday 1:30-3:30PM: Conflict with ITS325, MAS210
 - Tuesday 10:30-11:30AM: Conflict with GTS211
 - Let's add
 - **Monday 9:00-10:30**
 - I'm not limited to these time slots.
 - Usually in my office (BKD3601-7) from 8AM-5PM
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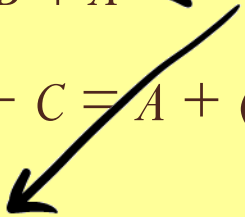
Today

- Some of us participate in the SIIT day activities.
- So, the lecture today will contain no new material.
 - We will do a lot of examples.
- These slides will be posted on the course web site later today.
 - Some of them are the same as what you have as hardcopy

- At 9:30, we will have attendance check
 - For those participate in SIIT D.A.,
Provide evidence to get checked.

Example

The associative law for addition is normally written as

- a. $A + B = B + A$ ← Commutative
 - b. $(A + B) + C = A + (B + C)$ ← Associative
 - c. $AB = BA$
 - d. $A + AB = A$ ← Covering
- 

Example

The Boolean equation $AB + AC = A(B + C)$ illustrates

a. the distribution law

b. the commutative law $\leftarrow x + y = y + x$, $xy = yx$

c. the associative law $\leftarrow x + (y + z) = (x + y) + z$

d. DeMorgan's theorem

Example

The Boolean expression $A \cdot 1$ is equal to

a. A

b. B

c. 0

d. 1

Example

The Boolean expression $A + 1$ is equal to

- a. A
- b. B
- c. 0
- d. 1

Example

The Boolean equation $AB + AC = A(B + C)$ illustrates

- a. the distribution law
- b. the commutative law
- c. the associative law
- d. DeMorgan's theorem

Review: Three Useful Rules

These rules do not exist in elementary algebra

$$\textcircled{1} \quad (\underline{A+B})(\underline{A+C}) = A + BC$$

$$\textcircled{2} \quad A + AB = A$$

$$= A \cdot 1 + A \cdot B$$

$$= A(1+B) = A \cdot 1 = A$$

Examples:

$$\checkmark \quad \boxed{XY} + \boxed{XYZ} = \boxed{XY}$$

$$W\bar{X}Y + WY = WY$$

$$= \bar{X}\boxed{WY} + \boxed{WY}$$

$$= \boxed{} = WY$$

$$\textcircled{3} \quad A + \bar{A}B = A + B$$

$$= A + \underbrace{AB + \bar{A}B}$$

$$= A + B(A + \bar{A}) = A + B \cdot 1$$

$$= A + B$$

Examples:

$$\bar{A} + AB = \bar{A} + B$$

$$XY + \bar{X}YZ = XY + Z$$

$$\boxed{\bar{A}} + \boxed{A}(\bar{B} + \bar{C}) = \bar{A} + \underline{\underline{\bar{B} + \bar{C}}}$$

Principle of Duality

Any theorem or identity remains true if $0 \leftrightarrow 1$

Example: $\cdot \leftrightarrow +$

$$X + 1 = 1 \qquad X + \bar{X} = 1$$

$$X \cdot 0 = 0 \qquad X \cdot \bar{X} = 0$$

Caution: $X + (X \cdot Y) = X \longrightarrow X \cdot (X + Y) = X$

- Parenthesize an expression fully before taking its dual!

$$\begin{array}{l} \longrightarrow (X \cdot Y) + (X \cdot Z) = X \cdot (Y + Z) \quad (X + Y)(X + Z) \\ \longrightarrow (X + Y) \cdot (X + Z) = X + Y \cdot Z \end{array}$$

factoring

Duality Principle in Action

(T1)	$X + 0 = X$	(T1')	$X \cdot 1 = X$	(Identities)
(T2)	$X + 1 = 1$	(T2')	$X \cdot 0 = 0$	(Null elements)
(T3)	$X + X = X$	(T3')	$X \cdot X = X$	(Idempotency)
(T4)	$(X')' = X$			(Involution)
(T5)	$X + X' = 1$	(T5')	$X \cdot X' = 0$	(Complements)

(T6)	$X + Y = Y + X$	(T6')	$X \cdot Y = Y \cdot X$	(Commutativity)
(T7)	$(X + Y) + Z = X + (Y + Z)$	(T7')	$(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)$	(Associativity)
(T8)	$X \cdot Y + X \cdot Z = X \cdot (Y + Z)$	(T8')	$(X + Y) \cdot (X + Z) = X + Y \cdot Z$	(Distributivity)
(T9)	$X + X \cdot Y = X$	(T9')	$X \cdot (X + Y) = X$	(Covering)
(T10)	$X \cdot Y + X \cdot Y' = X$	(T10')	$(X + Y) \cdot (X + Y') = X$	(Combining)

Example

Using Boolean algebra, simplify

space

$$\overline{ABC} + \overline{(A + B + \overline{C})} + \overline{A}\overline{B}\overline{C}D$$

← DeMorgan's Theorem

$$= \overline{A}\overline{B}C + \overline{A} \cdot \overline{B} \cdot C + \overline{A}\overline{B}\overline{C}D$$

$$= \overline{A}\overline{B}C + \overline{A}\overline{B}\overline{C}D$$

$$= \overline{A}\overline{B}(C + \overline{C}D) = \overline{A}\overline{B}(C + D) = \overline{A}\overline{B}C + \overline{A}\overline{B}D$$

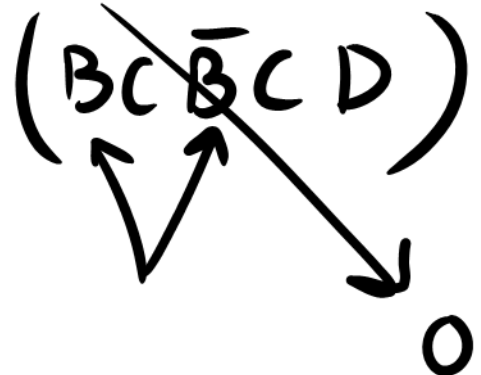
Example

Show that

$$\begin{aligned} & \underbrace{(A+B)(A+C)(A+D)}_{= A+BCD} \\ &= (A+BC)(A+D) \\ &= (A+\triangle)(A+\square) \\ & \quad \quad \quad \uparrow \quad \quad \quad \uparrow \\ & \quad \quad \quad BC \quad \quad \quad D \\ &= A + \triangle \cdot \square = A + BCD \quad \# \end{aligned}$$

Example

Using Boolean algebra, simplify

$$\begin{aligned} & (B + BC)(B + \overline{B}C)(B + D) \\ &= B + (BC\overline{B}CD) \\ &= B \end{aligned}$$


Example

Using Boolean algebra, simplify

$$ABCD + AB(\overline{CD}) + (\overline{AB})CD$$

$$AB(CD + \overline{CD})$$

1

$$AB$$

AB

$$+ \overline{AB}CD$$

$$= AB + CD$$

Example

Using Boolean algebra, simplify

$$ABC \left(AB + \overline{C} (\underline{BC} + \underline{AC}) \right)$$

~~$$\overline{C} \cdot C \cdot (B + A)$$

0~~

$$= \overbrace{ABC \cdot AB} = ABC$$

(Quiz # 1 IT)

Example

K-map

Using Boolean algebra, simplify

$$\overline{A}BC + \overline{A}BC + \overline{A}BC + \overline{A}BC + A\overline{B}C$$

$$\overline{A}BC + \overline{B} \left(\underbrace{AC + \overline{A}C}_{c(A + \overline{A})} + \underbrace{\overline{A}\overline{C} + A\overline{C}}_{\overline{C}(\overline{A} + A)} \right)$$

$$c \underbrace{(A + \overline{A})}_1$$

$$\overline{C} \underbrace{(\overline{A} + A)}_1$$

$$= \overline{A}BC + \overline{B} \underbrace{(c + \overline{C})}_1 = \overline{A}BC + \overline{B} = B(\overline{A}C) + \overline{B} = \overline{B} + \overline{A}C$$

Example

Using Boolean algebra, simplify

HW # 2
already
posted

$$\overline{A} + A\overline{B} + AB\overline{C}$$

$$\overline{A} + \overline{B} + AB\overline{C}$$

$$= \overline{B} + B(A\overline{C})$$

$$= \overline{B} + A\overline{C}$$

$$= \overline{A} + \overline{B} + A\overline{C} = \overline{B} + \overline{A} + \overline{C}$$

$$= \overline{A} + \overline{B} + \overline{C} = \overline{A \cdot B \cdot C}$$

Product Term

A single literal or a product of two or more literals.

Example: $A \cdot \bar{B} \cdot C$ ←

$$A \cdot C$$

$$\rightarrow A$$

$$A \cdot \bar{B} \cdot C \cdot D$$

$$\bar{A} \cdot \bar{B} \cdot \bar{C}$$

Caution:

$A \cdot B \cdot C$ is not a product term.

Q: When does $A \cdot \bar{B} \cdot C = 1$?

$$\left. \begin{array}{l} A = 1 \\ \bar{B} = 1 \\ C = 1 \end{array} \right\}$$

$A = 1$
$B = 0$
$C = 1$

$$(A, B, C) = (1, 0, 1)$$

Example

Find the value of X for all possible values of the variables when

$$\textcircled{101} \quad 011 \quad 001 \quad 000 \quad 100$$

$$X = ABC + \overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC} \quad \leftarrow \text{Standard sum of products}$$

	A	B	C						
1	0	0	0	1	0	0	0	1	0
1	0	0	1	1	0	0	1	0	0
0	0	1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	0	0	0
1	1	0	0	1	0	0	0	0	1
1	1	0	1	1	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0

Sum of products

Example

Find the value of X for all possible values of the variables when

